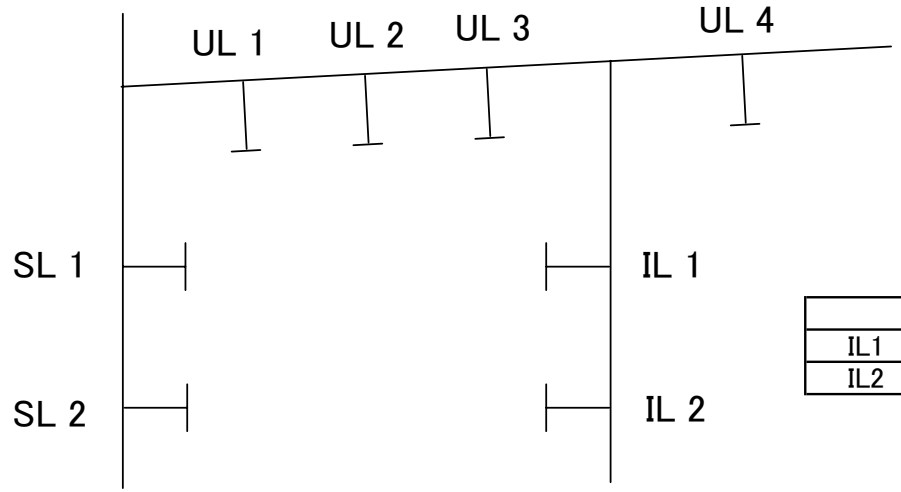
KC#1021 Question

Example of f2 in normal ballast condition for Zone M, stress range combination factor corresponding stress range due to horizontal bending moment

	y/B	f2
UL 1	0.49	0.24
UL 2	0.47	0.22
UL 3	0.46	0.21
UL 4	0.44	0.19



	z/D	f2
SL1	0.97	0.18
SL2	0.94	0.21

	z/D	f2
IL1	0.97	0.49
IL2	0.94	0.47

KC#1021 Answer

Stress Combination for Fatigue Analysis of Ship Structures

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A methodology for calculating the correlation factors to combine the long-term dynamic stress components of ship structure from various loads in seas is presented. The proposed methodology is valid for a stationary ergodic narrow-banded Gaussian process. The total combined stress in short-term sea states is expressed by linear summation of the component stresses with the corresponding combination factors. This expression is proven to be mathematically exact when applied to a single random sea. The long-term total stress is similarly expressed by linear summation of component stresses with appropriate combination factors. The stress components considered here are due to wave-induced vertical bending moment, wave-induced horizontal bending moment, external wave pressure, and internal tank pressure. For application, the stress combination factors are calculated for longitudinal stiffeners in midship cargo and ballast tanks of a crude oil tanker. It is found that the combination factors strongly depend on wave heading and period in the short-term sea states. It is also found that the combination factors are not sensitive to the selected probability of exceedance level of the stress in the long-term sense.
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Introduction

Ship structures are subjected to various types of loads during voyages. The loads include wave-induced dynamic load, hydrostatic load, transient impact-slamming load, sloshing load, thermal load, and so on. For design, strength evaluation, and fatigue analysis of the ship structures, correlation of the various load-stress components should be properly taken into account. In the design and strength analysis by finite element analysis, the load cases are determined by identifying the dominant load parameters. For each load case, the dominant load response is maximized at a specific wave-heading angle, and the design wave period and height at which the response is at the maximum are determined. Then the load combination factors representing the phase correlation between the dominant load response and secondary load response are determined. This is the so-called regular wave approach, which uses the instantaneous response concept. The load combination factors are basically calculated from transfer functions and phase angles between the dominant and secondary load responses for each load case. This regular wave approach has been widely used in the local scantling and finite element analysis of ship structures [1]. In fatigue analysis of ship structures, however, the long-term stresses rather than instantaneous ones are of main interest. Therefore, an irregular wave approach is more appropriate for combining fatigue load-stress components than the regular wave approach. Various types of load-stress combination methods using the irregular wave approach can be found in [2–6]. In this paper, a consistent and complete method for the combination factors in multiple sea states is presented.

The structural members around fatigue-sensitive locations are subjected to loadings attributed to multiple load effects. Since fatigue is a process of cyclic accumulation of damage in a structure, the cyclic loadings are considered important for fatigue assessment of ship structures. We consider hull girder loads (e.g., vertical and horizontal bendings), external wave pressure, and internal tank pressure resulting from ship motion. These are relatively high-cycle loads that induce the fatigue and occur in a structure in the range of elastic deformation. Other cyclic loadings, such as impact-slamming or low-cycle loads, which may result in significant levels of stress ranges over the expected lifetime of the vessel, are not considered here.

In this paper, we propose a methodology for calculating the stress combination factors, properly accounting for the correlation of the fatigue stress components. The proper combination of stress components is important to derive the total stress values for accurate evaluation of fatigue life. The stress combination factor, in short, represents the relationship between the total stress and each of the stress components. The combination factor should properly take into account the phase correlation between the total stress and each component stress. Here, the total stress at the specific structural location is expressed by linear summation of component stresses with the combination factors. The mathematical formulation is based on an assumption of a stationary ergodic narrow-banded Gaussian process. The formulation can be proven to be mathematically exact when applied to a single random sea. To determine the combination factors in the long-term sense, a generalization procedure for the correlation of extreme values at a given probability of exceedance is necessary. We use the calculated percentage probability of contribution for each scatter diagram entry as a weighting factor to obtain an appropriately weighted value of the combination factors.

For application of the proposed methodology here, the direct calculations of the combination factors are performed for longitudinal stiffeners in midship cargo and ballast tanks of a crude oil tanker. The longitudinals investigated are on the outer bottom, outer side-shell, inner bottom, inner side-shell, deck, and longitudinal bulkhead. The stresses considered here are due to four load components (i.e., wave-induced vertical bending moment, wave-induced horizontal bending moment, external wave pressure, and internal tank pressure). The analysis results show that the combination factors are strongly dependent on wave heading and period in short-term sea states. The combination factors in the long-term sense are also investigated depending on the probability of the exceedance level of the stress value. It is found that the stress combination factors are not dependent on the selected probability level.

Stress Transfer Function

The component stochastic analysis can be used to calculate the stress transfer function at a particular structural location. The transfer function of the stress due to each load component is de-

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termined from the load transfer function and stress factor. To calculate the stress transfer function, the corresponding stress factor is multiplied to the load transfer function. The load transfer functions, which are a function of vessel speed, wave heading angle, and wave frequency are the typical outputs of the seakeeping analysis program. The stress factor can be calculated through particular structural analysis techniques, which can be either simple beam theory or finite element analysis procedures. The sophistication of the structural analysis needed depends on the physical system to be analyzed and the type of structural detail and type of structural loading considered. For our application, the stress factors are calculated by the simple beam theory.

The transfer function of the total stress is obtained by simply adding the transfer functions of the component stresses. Therefore, a set of the stress transfer functions can be generated at the vessel speeds, wave headings, and wave frequencies under consideration. The response spectra for the stress transfer functions can then be determined for a given wave spectrum. Summing the stress distributions for the various sea states in the scatter diagram, the long-term distribution of dynamic stresses can be obtained.

Stress Combination Method for a Short-Term Sea State

Consider a one-component stress effect, such as a longitudinal stress in a particular detail. This stress component will receive contributions from several different mechanisms that are based on the response of the overall structure to random waves. We assume contributions arise from four mechanisms, $i=1,2,3,4$, where $i=1$ corresponds to vertical bending, $i=2$ to horizontal bending, $i=3$ to external wave pressure, and $i=4$ to internal tank pressure due to accelerations of fluid in a tank. It is assumed that the stress responses in irregular waves are stationary ergodic narrow-banded Gaussian processes.

Given an input wave spectrum, the relationship between the input spectrum $S_x(\omega)$ and the output (response) spectrum $S_y(\omega)$ for a single component is given by the following equation:

$$S_y(\omega) = H_i(\omega)H_i^*(\omega)S_x(\omega) = |H_i(\omega)|^2 S_x(\omega) \tag{1}$$

where $|H_i(\omega)|$ is the transfer function of the response, e.g., stress transfer function and the superscript * denotes a complex conjugate. The variance (zeroth moment) of a response spectrum is obtained by integrating the spectrum over all encounter frequencies, so obtaining

$$\sigma_c^2 = m_0 = \int_0^\infty S_y(\omega)d\omega = \int_0^\infty |H_i(\omega)|^2 S_x(\omega)d\omega \tag{2}$$

The variance (zeroth moment) of a response spectrum comprising four contributions can be determined according to [6]

$$\sigma_c^2 = m_0 = \int_0^\infty S_y(\omega)d\omega = \sum_{i=1}^{4}\int_0^\infty |H_i(\omega)|^2 S_x(\omega)d\omega + \sum_{i=1}^{4}\sum_{j=1}^{4}{}_{i\neq j}\int_0^\infty H_i(\omega)H_j^*(\omega)S_x(\omega)d\omega \tag{3}$$

Equation (3) can be reexpressed in terms of the variances and correlation coefficients ρ_{ij} of the different contributing components in the form

$$\sigma_c^2 = \sum_{i=1}^{4}\sigma_i^2 + \sum_{i=1}^{4}\sum_{j=1}^{4}{}_{i\neq j}\rho_{ij}\sigma_i\sigma_j \tag{4}$$

where

$$\sigma_i^2 = \int_0^\infty |H_i(\omega)|^2 S_x(\omega)d\omega \tag{5}$$

and

$$\rho_{ij} = \frac{1}{\sigma_i\sigma_j}\int_0^\infty \mathrm{Re}[H_i(\omega)H_j^*(\omega)]S_x(\omega)d\omega \tag{6}$$

Alternatively, ρ_{ij} can be expressed in the form

$$\rho_{ij} = \frac{1}{\sigma_i\sigma_j}\int_0^\infty |H_i(\omega)||H_j(\omega)|(\cos(\phi_j(\omega)-\phi_i(\omega))S_x(\omega)d\omega \tag{7}$$

where ϕ is the phase angle. Writing Eq. (4) explicitly, we have

$$\sigma_c^2 = \sigma_1^2+\sigma_2^2+\sigma_3^2+\sigma_4^2+2\rho_{12}\sigma_1\sigma_2+2\rho_{13}\sigma_1\sigma_3+2\rho_{14}\sigma_1\sigma_4 + 2\rho_{23}\sigma_2\sigma_3+2\rho_{24}\sigma_2\sigma_4+2\rho_{34}\sigma_3\sigma_4 \tag{8}$$

Alternatively, by generalization of the equations and illustration given in Appendixes A and B, this can be recast as for the combinations of the transfer function in an entirely equivalent form

$$\sigma_c = \rho_{c1}\sigma_1+\rho_{c2}\sigma_2+\rho_{c3}\sigma_3+\rho_{c4}\sigma_4 \tag{9}$$

where ρ_{c1}, ρ_{c2}, ρ_{c3}, and ρ_{c4} are the short-term combination factors. The above results can be generalized to include the direction of ship heading relative to predominant wave direction α and wave-spreading angle μ by using the following expressions:

$$\sigma_i^2(\alpha) = \int_{-\pi/2}^{\pi/2}\int_0^\infty |H_i(\omega,\alpha-\mu)|^2 S_x(\omega,\mu)d\omega d\mu \tag{10}$$

and

$$\rho_{cj}(\alpha) = \int_{-\pi/2}^{\pi/2}\frac{1}{\sigma_c\sigma_j}\int_0^\infty \mathrm{Re}[H_c(\omega,\alpha-\mu) H_j^*(\omega,\alpha-\mu)]S_x(\omega,\mu)d\omega d\mu \tag{11}$$

Since the heading angle is to be represented as uniformly distributed between 0 and 360 deg, the outer integral will be represented as a sum over the specified heading angles divided by number of wave-heading angles considered.

Thus far, the above discussion has been in reference to characteristic values and correlations for response to a single sea state characterized by a given spectrum. This is normally termed as the short-term response. During the course of its design life, the vessel will encounter a large number of spectra with different characteristic values of significant wave height and period and these will be encountered at a range of directions with respect to the vessel's forward speed. The vessel may also be loaded to different levels of draft for significant fractions of its design life.

Stress Combination Method for Long-Term Multiple Sea States

The long-term environment is characterized by a wave scatter diagram that specifies the relative numbers of each sea state that might be experienced over a long period and some rules for the distribution of headings of waves relative to the vessel and the portions of its life in each identified load condition between the waves and the ship. The combined stress associated with a long-term distribution of stress might be calculated from an equation similar in format to that derived for a short-term sea state, namely,

$$\sigma_c^* = C_1\sigma_1^*+C_2\sigma_2^*+C_3\sigma_3^*+C_4\sigma_4^* \tag{12}$$

which is generalized to incorporate four contributing components. Here, σ_c^* is the characteristic value of combined stress at some life-time (probability of exceedance level); σ_1^*, σ_2^*, σ_3^*, and σ_4^* are the characteristic value of the stress component 1 (due to vertical bending), stress component 2 (due to horizontal bending), stress component 3 (external wave pressure), stress component 4 (internal tank pressure), respectively, at the same probability of exceedance level. The long-term correlation coefficients C_1, C_2, C_3, and C_4 are referred to as the stress combination factors for combining long-term responses. For example, C_1 is related to correlation co-

efficient between combined stress and component 1, and C_2 is related to correlation coefficient between combined stress and component 2. The procedure for determining the combination factors corresponding to a particular probability of exceedance across a combined scatter-diagram-heading distribution of sea states is described below.

For each entry in each scatter diagram for each heading, the zeroth moment of the (short-term) spectral response is determined. Also, we determine the second moment of the (short-term) spectral response, bearing in mind that this value is a function of wave encounter frequency. For the scatter diagram entry associated with each heading, significant wave height and zero-crossing period (of the waves) can be used to calculate the zero-crossing period of the response and hence the number of response cycles. We normalize these values by dividing by the total number of response cycles for all headings and scatter diagram entries.

The contribution that any one scatter-diagram-heading contribution makes to the long-term exceedance distribution of the response is then the sum of Rayleigh distributions multiplied by the normalized number of response cycles, so that the long-term probability that the response will exceed a particular value x is calculated from $\Sigma_k(n_k/n_{\text{total}})p_k \exp(-x^2/2/m_{0k})$, where the sum over k is over the entire set of scatter diagrams of significant wave height, zero-crossing wave period and heading contributions, n_k is the number of stress cycles that will be experienced for each scatter-diagram entry at each heading, n_{total} is the total number of cycles for the entire lifetime summed over all scatter diagram entries and headings, p_k is the probability of occurrence from the wave scatter table, and m_{0k} is the corresponding zeroth moment of the spectral response. The values of x_N that make this expression equal to 10^{-N} are those corresponding to this long-term probability of exceedance

$$\sum_k \frac{n_k}{n_{\text{total}}} p_k \exp\left(-\frac{x_N^2}{2m_{0k}}\right) = \frac{1}{10^N} \tag{13}$$

Once a value of x_N is determined, then substituting this value back into each separate term in the above summation and multiplying by $10^{(N+2)}$ gives the percentage contribution that each scatter-heading entry makes to the 10^{-N} level of exceedance probability. This procedure can be applied to the combined stress or any component stress to determine the relevant component probabilities at any exceedance level for each of the components.

The percentage component probabilities contributing to the combined stress (i.e., the right-hand side of Eq. (12)) have been applied as weighting factors to the calculated correlation coefficients to determine weighted average values of the correlation coefficient that apply to the responses at the specified level of exceedance. That is, the stress combination factor in the long-term sense can be obtained from

$$C_j = \sum_k w_k \rho_{cj}, \quad j = 1,2,3,4 \tag{14}$$

where ρ_{cj} is the stress combination factor in the short-term sense (single seas state); w_k is the weighting factor to derive the stress combination factor in long-term sense, which can be expressed by

$$w_k = 10^N \frac{n_k}{n_{\text{total}}} p_k \exp\left(-\frac{x_N^2}{2m_{0k}}\right) \tag{15}$$

Though not as precise, the alternative expressions for the weighting factor

$$w_k = 10^N p_k \exp\left(-\frac{x_N^2}{2m_{0k}}\right) \tag{16}$$

may be used. It is found that there is no significant difference in the stress combination factors C_j between application of the two weighting methods. This will be discussed further in a following section. Finally, the right-hand side of the postulated Eq. (14) can be evaluated for any exceedance level for each component stress and the weighted averages of the combination factors as specified above.

Table 1 Principal dimensions of 298,000 DWT class crude oil tanker

Length, B. P. (m)	316.0
Length, Scant. (m)	317.69
Breadth, Mld. (m)	60.0
Depth, Mld. (m)	29.7
Draught, Mld. (design) (m)	19.2
Block coeff. (full load)	0.810

Results and Discussions

The stress combination factors have been calculated for longitudinal stiffeners in midship tanks (cargo and ballast tanks) of a 298,300 DWT class crude oil tanker with two different loading conditions (full load and normal ballast). The principal dimensions of the subject vessel are summarized in Table 1.

Figure 1 shows the schematic sketch of the midship section with locations of longitudinal stiffeners for the subject vessel. First, the load transfer functions are obtained by a seakeeping analysis with unit amplitude waves for the combinations of loading conditions, wave headings, and wave frequencies. These calculations are performed using PRECAL [7], a three-dimensional panel code for analyzing the wave-induced motions and loads of the ship in six degrees of freedom. PRECAL has been developed based on linear wave-motion assumptions and a boundary element implementation of three-dimensional forward-speed diffraction-radiation theory. The real and imaginary values of vertical and horizontal bending moments, external wave pressure, and acceleration components at the center of gravity of the tanks are obtained. The load transfer function is then multiplied by the corresponding stress factor to obtain the stress transfer function. The transfer functions of the internal pressure are obtained from the acceleration components of the tank. The vessel speed used is 75% of the design speed, which is 11.25 kn. In present study, the prescribed scatter diagram of significant wave heights and wave periods is the IACS Recommendation No. 34 scatter diagram for trading in the North Atlantic. We use the Pierson-Moskowitz wave spectrum with a cosine-squared spreading function to represent a short-crested wave energy spectrum in the short-term calculations

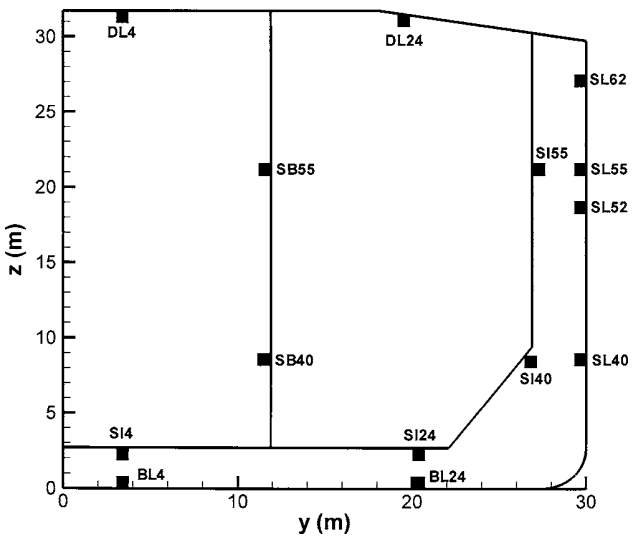

Fig. 1 Sketch of midship section with longitudinal locations for 298,300 DWT crude oil tanker

Table 2 Combination factors and long-term stress values for longitudinal stiffeners in full load condition

Member	σ_{VBM}	C_1	σ_{HBM}	C_2	σ_{PEX}	C_3	σ_{PIN}	C_4	σ_{TOT}
BL4	55.83	0.97	2.58	0.40	10.83	0.60	0	NA	61.86
BL24	55.83	0.87	15.47	0.56	23.26	0.41	0	NA	66.66
SL40	18.27	0.22	22.44	0.78	50.41	0.88	0	NA	65.82
SL52	28.00	0.20	22.49	0.63	56.73	0.93	0	NA	72.59
SL55	39.57	0.32	22.50	0.55	49.85	0.87	0	NA	68.40
SL62	66.56	0.59	22.61	0.31	47.25	0.65	0	NA	77.41
DL4	84.05	1.00	2.58	−0.33	0	NA	0	NA	83.33
DL24	81.16	1.00	14.81	−0.21	0	NA	0	NA	77.77
SI4	47.29	0.98	2.58	0.51	0	NA	18.69	−0.48	38.70
SI24	47.29	0.87	15.47	0.69	0	NA	20.02	−0.32	45.49
SI40	18.88	0.34	20.43	0.86	0	NA	21.20	0.42	32.87
SI55	39.57	0.77	20.85	0.15	0	NA	20.72	0.78	49.85
SB40	18.27	0.82	8.71	0.43	0	NA	14.07	0.52	26.08
SB55	39.57	0.90	8.77	−0.25	0	NA	23.90	0.19	37.83

and equal probability of occurrence of each heading in long-term calculations. Heading of the vessel relative to the waves is assumed to be uniformly distributed between 0 (following seas) and 360 deg with a 30 deg interval.

The stress combination factors and long-term extreme values of stresses at end longitudinal connections for the two different load conditions are obtained from direct calculations, as shown in Tables 2 and 3. All heading contributions are considered in the calculations. The probability of occurrence from the wave scatter table is used as the weighting factor (based on Eq. (16)) to calculate the stress combination factors. As can be seen in the Tables 2 and 3, the combination factors can be much different, even at the same longitudinal location, depending on loading conditions. Here, σ_{VBM}, σ_{HBM}, σ_{PEX}, σ_{PIN}, and σ_{TOT} are the long-term values of stress amplitudes, at the probability level of 10^{-4} due to vertical bending moment, horizontal bending moment, external wave pressure, internal tank pressure, and combined total, respectively. It should be noted that the unit of the stress values is in MPa.

Figure 2 shows the combination factor C_3 related to external wave pressure over wave scatter-diagram entry, i.e., significant wave height $H_{1/3}$ and average zero-crossing wave period T_z. The subject vessel is in ballast condition, the wave heading is 150 deg and the longitudinal SL40 investigated is on the outer-side shell. The results indicate that the combination factor C_3 is independent of the wave height and varies only with the zero-crossing wave period. It is seen in Fig. 3 that the stress combination factor C_3 is a function of wave heading and period. It is found that these observations are also valid for other combination factors C_1, C_2, and C_4.

Figure 4 shows the comparison of the stress combination factor C_1 at the two different probability levels, i.e., 10^{-4} and 10^{-8}. The subject vessel is in full load condition and all wave-heading contributions are considered. The probability of occurrence from the wave scatter table using Eq. (16) is used as the weighting factor. As can be seen in Fig. 4, the combination factors are not much different, depending on the selected probability levels. This observation is valid for all probability levels and similar to the one obtained from a different type of the load combination method as proposed in [8].

The dependence of the combination factor on the different weighting methods is also investigated. The subject vessel is in full load condition, and the probability level is 10^{-4}. The two different weighting methods are used to calculate the combination factor of the extreme value at the given probability of exceedance level; one is based on the number of stress cycles (denoted as "Response" using Eq. (15)), and the other is based on the probability of occurrence from the wave scatter table (denoted as "Wave" using Eq. (16)). As can be seen from the results of the combination factor C_1 in Fig. 5, the combination factors show no significant dependence on the selected weighting method.

Conclusions

A methodology for determining the stress combination factor for the fatigue analysis of ship structures is proposed. The methodology is based on the irregular wave approach that involves the short- and long-term direct calculations. The total stress at the specific structural location is expressed by linear summation of component stresses with the combination factors that consider the phase correlation between the component stress and the total stress. The formulation was mathematically proven to be exact in the short-term sense based on an assumption of a stationary ergodic narrow-banded Gaussian process. A generalization procedure for determining the combination factors of long-term ex-

Table 3 Combination factors and long-term stress values for longitudinal stiffeners in ballast condition

Member	σ_{VBM}	C_1	σ_{HBM}	C_2	σ_{PEX}	C_3	σ_{PIN}	C_4	σ_{TOT}
BL4	51.78	0.95	1.10	0.61	8.13	0.75	20.35	0.84	73.27
BL24	51.78	0.85	6.57	0.57	20.13	0.53	33.16	0.69	81.17
SL40	16.95	0.63	9.53	0.44	29.15	0.55	30.65	0.65	50.80
SL52	25.97	0.15	9.55	−0.02	0	NA	29.57	0.83	28.12
SL55	36.70	0.61	9.55	−0.28	0	NA	26.87	0.39	30.23
SL62	61.73	0.87	9.60	−0.52	0	NA	37.04	0.13	53.40
DL4	77.95	1.00	1.10	−0.65	0	NA	0	NA	77.37
DL24	75.27	1.00	6.29	−0.62	0	NA	0	NA	71.97
SI4	43.86	0.96	1.10	0.63	0	NA	15.10	0.86	55.99
SI24	43.86	0.90	6.57	0.61	0	NA	24.76	0.74	61.78
SI40	17.51	0.77	8.68	0.55	0	NA	26.69	0.80	39.52
SI55	36.70	0.86	8.85	−0.40	0	NA	20.73	−0.01	27.82
SB40	16.95	0.97	3.70	0.73	0	NA	0	NA	19.17
SB55	36.70	1.00	3.72	−0.61	0	NA	0	NA	34.76

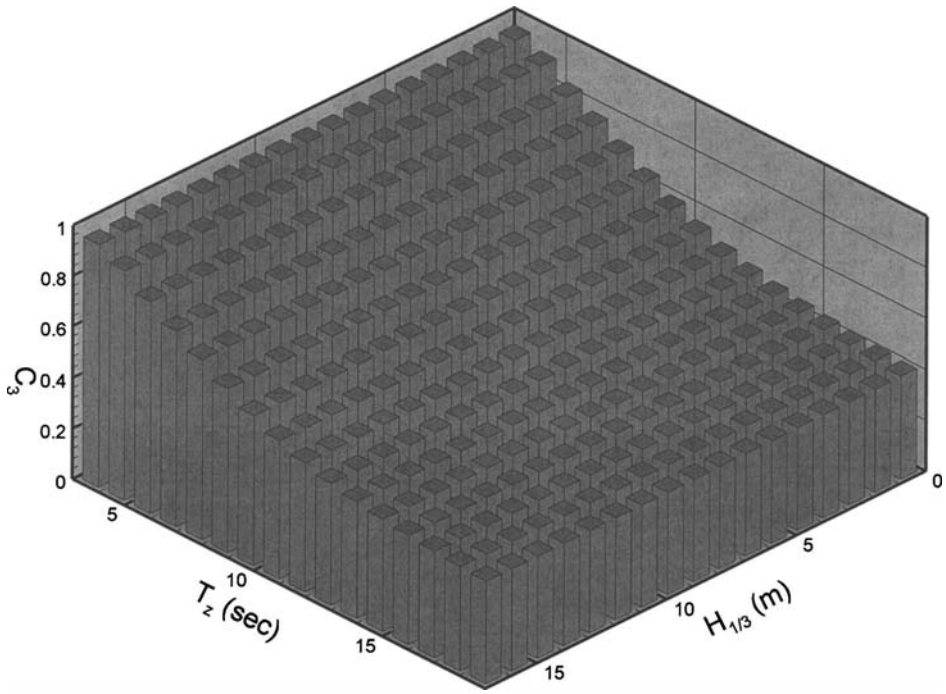

Fig. 2 Combination factor C_3 over wave scatter entry

treme values at the given probability of exceedance level was introduced. It is proposed to use the calculated percentage probability of contribution for each scatter diagram entry as a weighting factor to obtain an appropriately weighted value of the combination factors obtained in short-term sea states.

The direct calculations of the stress combination factors were then performed for end longitudinal connections in midship cargo and ballast tanks of a crude oil tanker in full load and ballast conditions. The results show, in principle, that the combination factors strongly depend on structural location, cargo loading condition, wave heading, and average zero-crossing wave period. It is found that the combination factors for long-term responses in multiple sea states are not sensitive to the selected probability level and the weighting method.

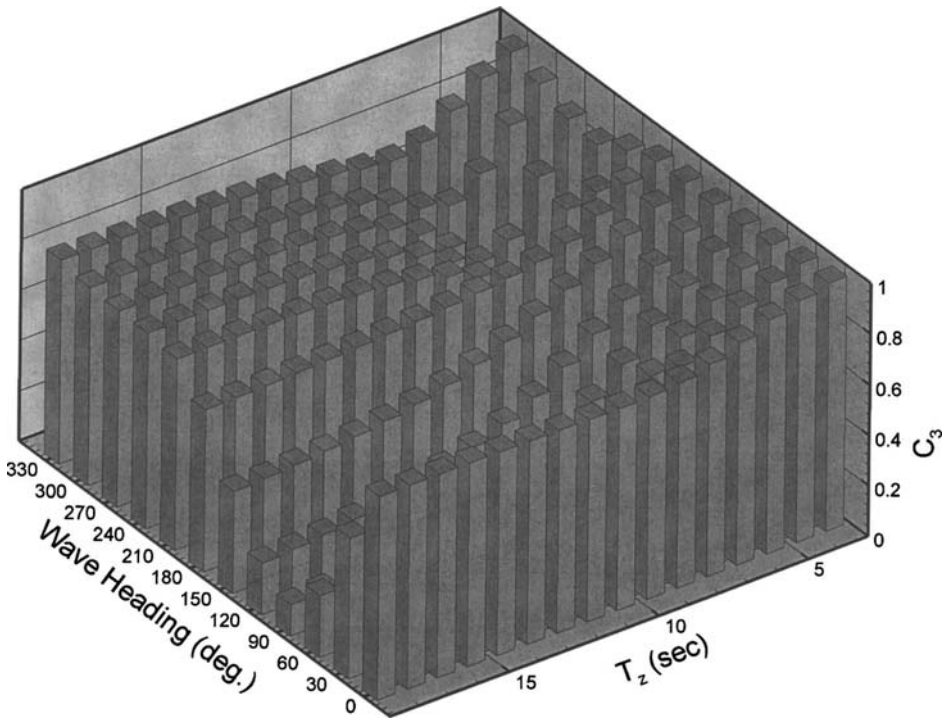

Fig. 3 Combination factor C_3 over wave heading and period

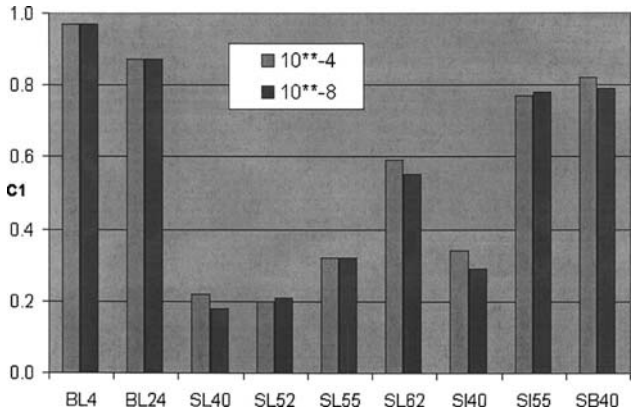

Fig. 4 Combination factor C_1 at probability levels 10^{-4} and 10^{-8}

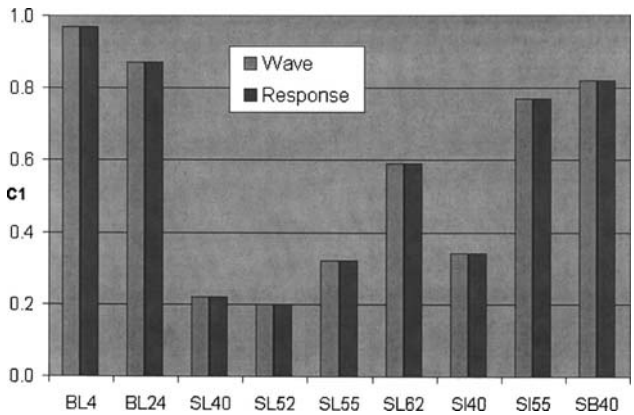

Fig. 5 Combination factor C_1 with different weighting methods

Acknowledgments

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Appendix A: Equivalence of the Two Expressions for Combination of Transfer Functions

Consider two transfer functions, H_1 and H_2, and the combined transfer function H_c. The accompanying sketch, shown in Fig. 6, and the algebra below demonstrate the equivalence of the two expressions for combining transfer functions. The two expressions are as follows:

$$|H_c|^2 = |H_1|^2 + |H_2|^2 + 2|H_1||H_2|\cos(\phi_2 - \phi_1) \tag{A1}$$

which is the normal expression for the third side of a triangle, given two sides and an included angle. If the direction of the resultant is known, an alternative expression is possible

$$|H_c| = |H_1|\cos(\phi_c - \phi_1) + |H_2|\cos(\phi_c - \phi_2) \tag{A2}$$

Note that ϕ_c is a function of H_1, H_2, and H_c. Geometrically, the equivalence is intuitively self-evident. The first expression (A1) is the normal result of vector addition obtained according to a "parallelogram" construction, as in parallelogram of forces. The second expression (A2) is visualized as the components of each of the two vectors, H_1 and H_2, resolved along the direction of the resultant. Since these are the only components that can contribute to the resultant, they must sum to the length of the resultant.

To demonstrate equivalence by algebra, express the cosines and sines of the angles involved in terms of the real parts A, the imaginary parts B, and the magnitudes. Then, expand the cosines in the second expression as follows:

$$\cos\phi_1 = A_1/|H_1|, \quad \sin\phi_1 = B_1/|H_1|$$

and similarly for H_2 and H_c

$$|H_c| = |H_1|\{A_c/|H_c| \cdot A_1/|H_1| + B_c/|H_c| \cdot B_1/|H_1|\} + |H_2| \times\{A_c/|H_c| \cdot A_2/|H_2| + B_c/|H_c| \cdot B_2/|H_2|\}$$

$$|H_c|^2 = A_cA_1 + B_cB_1 + A_cA_2 + B_cB_2$$

Substituting $A_c = A_1 + A_2$ and $B_c = B_1 + B_2$, we obtain

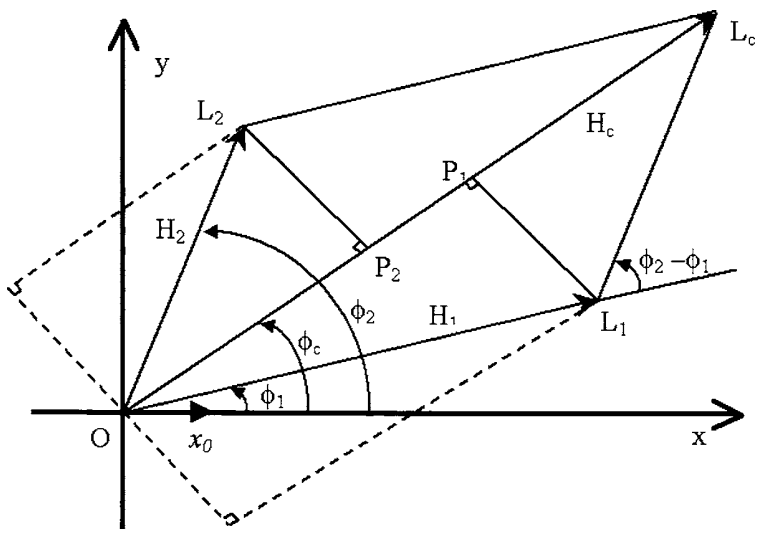

Fig. 6 Sketch for demonstration of equivalence of the two expressions for combining transfer functions

$$|H_c|^2 = |H_1|^2 + |H_2|^2 + 2(A_1A_2 + B_1B_2) = |H_1|^2 + |H_2|^2 + 2|H_1| \times |H_2|\cos(\phi_2 - \phi_1)$$

Similarly, for three components,

$$|H_c|^2 = |H_1|^2 + |H_2|^2 + |H_3|^2 + 2|H_1||H_2|\cos(\phi_2 - \phi_1) + 2|H_1| \times |H_3|\cos(\phi_3 - \phi_1) + 2|H_2||H_3|\cos(\phi_3 - \phi_2)$$

or

$$|H_c| = |H_1|\cos(\phi_c - \phi_1) + |H_2|\cos(\phi_c - \phi_2) + |H_3|\cos(\phi_c - \phi_3)$$

Note that these relationships apply between the transfer functions themselves, and that they are unchanged by multiplication, throughout, by a constant. They apply whether the transfer functions are being used on a deterministic signal to predict the output to a particular input time trace, such as a sinusoid of a particular frequency (in the simplest case), or to a spectral coordinate so that a spectral average value can be obtained.

Because a transfer function is frequency dependent, an identical set of relations will apply at each frequency. For any given spectrum, an average value can be obtained for any or all of the terms on the right-hand side of either equation. Note that it is the application of a spectrum that results in the loss of phase information, not the application of the above equations. Finally, note that each of the cosine terms can be cast as a correlation coefficient. Because in this case it is the common input of a wave train that causes the response, any lack of correlation among inputs or between an input and output signal is due to the phase shift caused by the transfer functions.

Appendix B: Alternative Forms for Combination of Transfer Functions

The equivalence of the two expressions for combining transfer functions can be illustrated with the aid of the vector representation of the transfer functions, as shown in Fig. 6. Here, x_0 represents a unit sinusoidal input, $x_0 \cos \omega t$. H_1 and H_2 represent transfer functions, and H_c is the transfer function obtained by combining H_1 and H_2.

To combine H_1 and H_2 represent them as amplitudes, OL_1, OL_2, and phases, and then either

i. Draw parallelogram $OL_1L_cL_2$, then apply triangle rule in O, L_1, and L_c to give

$$(OL_c)^2 = |H_c|^2 = (OL_1)^2 + (L_1L_c)^2 + (OL_1)(L_1L_c)\cos(\phi_2 - \phi_1)$$

$$|H_c|^2 = |H_1|^2 + |H_2|^2 + |H_1||H_2|\cos(\phi_2 - \phi_1)$$

or

ii. If ϕ_c can be determined, then draw perpendiculars from L_1 and L_2 onto OL_c,

$$OL_c = OP_1 + OP_2$$

$$|H_c| = |H_1|\cos(\phi_c - \phi_1) + |H_2|\cos(\phi_c - \phi_2)$$

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